Footing Reinforcing Design (F-2) - Includes Uplift Strength Check:

$f`_c := 3000\ psi$ $E := 29000\ ksi$ $\lambda := 1.0$ $F_y := 60\ ksi$

$W := 6\ ft$ $L := 4\ ft$ $T := 10\ in$ $C_{top} := 2 \cdot in$ $C_{bottom} := 3\ in$

$Base_W := 12\ in$ $Base_L := 12\ in$ $\rho_{concrete} := 145\ pcf$

Loading: $P_{Dead} := 1\ kip$ $P_{Soil} := 200\ psf$ $P_{Wind} := -7\ kip$

$$P_{Self} := \rho_{concrete} \cdot W \cdot L \cdot T + P_{Soil} \cdot W \cdot L - P_{Soil} \cdot Base_W \cdot Base_L = 7.5\ kip$$

Max Soil Bearing Pressure: $\sigma_{max} := \begin{bmatrix} 3000\ psf \\ 3000\ psf \\ 3000\ psf \end{bmatrix}$ $bearingCapacity := \sigma_{max} \cdot W \cdot L = \begin{bmatrix} 72 \\ 72 \\ 72 \end{bmatrix} kip$

ASD Load Combinations:

$$P_{asd} := \begin{bmatrix} P_{Dead} + P_{Self} \\ P_{Dead} + P_{Self} + P_{Wind} \\ 0.6 \cdot \left(P_{Dead} + P_{Self}\right) + 0.6 \cdot P_{Wind} \end{bmatrix} = \begin{bmatrix} 8.5 \\ 1.5 \\ 0.9 \end{bmatrix} kip$$

Uplift Resisting Weight:

$$W_r := \begin{bmatrix} P_{Dead} + P_{Self} \\ P_{Dead} + P_{Self} \\ 0.6 \cdot \left(P_{Dead} + P_{Self}\right) \end{bmatrix} = \begin{bmatrix} 8.5 \\ 8.5 \\ 5.1 \end{bmatrix} kip$$

Footing Reinforcing:

Reinforcing Top:

$Bar`_W := 4$   $A`_{s\_W} := A_{bar}\left(Bar`_W\right) = 0.2\ in^2$   $N`_{bars\_W} := 5$ $Bar_{Spacing}\left(W, N`_{bars\_W}\right) = 16.5\ in$

$Bar`_L := 4$   $A`_{s\_L} := A_{bar}\left(Bar`_L\right) = 0.2\ in^2$   $N`_{bars\_L} := 4$ $Bar_{Spacing}\left(L, N`_{bars\_L}\right) = 14\ in$

Reinforcing Bottom:

$Bar_W := 4$ $A_{s\_W} := A_{bar}\left(Bar_W\right) = 0.2\ in^2$ $N_{bars\_W} := 5$ $Bar_{Spacing}\left(W, N_{bars\_W}\right) = 16.5\ in$

$Bar_L := 4$ $A_{s\_L} := A_{bar}\left(Bar_L\right) = 0.2\ in^2$ $N_{bars\_L} := 4$ $Bar_{Spacing}\left(L, N_{bars\_L}\right) = 14\ in$

Steel Depths:

Top:

$$d`_W := C_{top} + D_{bar}\left(Bar`_L\right) + \frac{D_{bar}\left(Bar`_W\right)}{2} = 2.75\ in$$

$$d`_L := C_{top} + D_{bar}\left(Bar`_W\right) + \frac{D_{bar}\left(Bar`_L\right)}{2} = 2.75\ in$$

Bottom:

$$d_W := T - \left(C_{bottom} + D_{bar}\left(Bar_L\right) + \frac{D_{bar}\left(Bar_W\right)}{2}\right) = 6.25\ in$$

$$d_L := T - \left(C_{bottom} + D_{bar}\left(Bar_W\right) + \frac{D_{bar}\left(Bar_L\right)}{2}\right) = 6.25\ in$$

Steel Ratios:

$$SteelCheck(\rho_w) := \text{if}\left(\rho_w > 0, \text{if}\left(\rho_w > 0.0018, \text{“OK”}, \text{“Not Enough”}\right), \text{“NA”}\right)$$

Top:

$$\rho_{\_W\_top} := \frac{A`_{s\_W} \cdot N`_{bars\_W}}{W \cdot \left(T - d`_W\right)} = 0.0019 \qquad SteelCheck\left(\rho_{\_W\_top}\right) = \text{“OK”}$$

$$\rho_{\_L\_top} := \frac{A`_{s\_L} \cdot N`_{bars\_L}}{L \cdot \left(T - d`_L\right)} = 0.0023 \qquad SteelCheck\left(\rho_{\_L\_top}\right) = \text{“OK”}$$

Bottom:

$$\rho_{\_W\_bottom} := \frac{A_{s\_W} \cdot N_{bars\_W}}{W \cdot d_W} = 0.0022 \qquad SteelCheck\left(\rho_{\_W\_bottom}\right) = \text{“OK”}$$

$$\rho_{\_L\_bottom} := \frac{A_{s\_L} \cdot N_{bars\_L}}{L \cdot d_L} = 0.0027 \qquad SteelCheck\left(\rho_{\_L\_bottom}\right) = \text{“OK”}$$

Single Shear:

$$V_n\left(f`_c, b, d, \rho_w\right) := \begin{array}{|l} \text{if } \rho_w \ge 0.0018 \\ \quad V_{c1} \leftarrow 2 \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \cdot b \cdot d \\ \quad V_{c2} \leftarrow 8 \cdot \lambda \cdot \left(\rho_w\right)^{\frac{1}{3}} \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \cdot b \cdot d \\ \quad \text{return } min\left(\max\left(V_{c1}, V_{c2}\right), 5 \cdot \lambda \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \cdot b \cdot d\right) \\ \text{else} \\ \quad \text{return } 0\ kip \end{array}$$

$$V_{n\_W} := V_n\left(f`_c, W, d_W, \rho_{\_W\_bottom}\right) = 49.295\ kip \qquad \phi V_{n\_W} := 0.75 \cdot V_{n\_W} = 36.971\ kip$$

$$V_{n\_L} := V_n\left(f`_c, L, d_L, \rho_{\_L\_bottom}\right) = 32.863\ kip \qquad \phi V_{n\_L} := 0.75 \cdot V_{n\_L} = 24.648\ kip$$

Double Shear:

$$V_{c\_punching}\left(W, L, Base_W, Base_L, d\right) := \begin{array}{|l} \beta \leftarrow \frac{\max(W, L)}{min(W, L)} \\ b_{o\_w} \leftarrow \text{if}\left(Base_L + d \ge L, 0\ in, Base_W + d\right) \\ b_{o\_l} \leftarrow \text{if}\left(Base_W + d \ge W, 0\ in, Base_L + d\right) \\ b_o \leftarrow 2 \cdot b_{o\_w} + 2 \cdot b_{o\_l} \\ \text{if } b_o > 0\ in \\ \quad \alpha_s \leftarrow 40 \\ \quad \text{return } min\left(\begin{bmatrix} 4 \cdot \lambda \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \\ \left(2 + \frac{4}{\beta}\right) \cdot \lambda \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \\ \left(\frac{\alpha_s \cdot d}{b_o} + 2\right) \cdot \lambda \cdot \sqrt{\frac{f`_c}{psi}} \cdot psi \end{bmatrix}\right) \cdot b_o \cdot d \\ \text{else} \\ \quad \text{return } 0\ kip \end{array}$$

$$\phi V_{c\_punching} := 0.75 \cdot V_{c\_punching}\left(W, L, Base_W, Base_L, min\left(d_W, d_L\right)\right) = 74.97\ kip$$

Calculate Moment Capacity:

$$M_n\left(A_{bottom}, N_{bottom}, A_{top}, N_{top}, b, T, d, d`, F_y, f_c, E\right) :=$$

$$\beta_1 \leftarrow \text{if } f`_c \le 4000\ psi$$
$$\quad 0.85$$
$$\text{else if } f`_c \le 8000\ psi$$
$$\quad 0.85 - 0.05 \cdot \frac{\left(f`_c - 4000\ psi\right)}{1000\ psi}$$
$$\text{else}$$
$$\quad 0.65$$

$$A_s \leftarrow A_{bottom} \cdot N_{bottom}$$
$$A`_s \leftarrow A_{top} \cdot N_{top}$$
$$\rho \leftarrow \frac{A_s}{b \cdot d}$$
$$\rho` \leftarrow \frac{A`_s}{b \cdot \left(T - d`\right)}$$

$$\text{if } \rho \ge 0.0018$$
$$\quad \text{if } \rho - \rho` \ge \frac{\beta_1 \cdot 0.85 \cdot f`_c \cdot d`}{F_y \cdot d} \cdot \frac{87000\ psi}{87000\ psi - F_y}$$
$$\quad\quad c \leftarrow \frac{\left(A_s - A`_s\right) \cdot F_y}{0.85 \cdot \beta_1 \cdot b \cdot f`_c}$$
$$\quad\quad a \leftarrow \beta_1 \cdot c$$
$$\quad\quad M_n \leftarrow A`_s \cdot F_y \cdot \left(d - d`\right) + \left(A_s - A`_s\right) \cdot F_y \cdot \left(d - \frac{a}{2}\right)$$
$$\quad \text{else}$$
$$\quad\quad c \leftarrow \frac{\sqrt{9 \cdot E^2 \cdot A`_s{}^2 - 6000 \cdot E \cdot A_s \cdot F_y \cdot A`_s + 10200 \cdot b \cdot d` \cdot \beta_1 \cdot f`_c \cdot E \cdot A`_s + 1000000 \cdot A_s{}^2 \cdot F_y{}^2} - 3 \cdot E \cdot A`_s + 1000 \cdot A_s \cdot F_y}{1700 \cdot b \cdot \beta_1 \cdot f`_c}$$
$$\quad\quad a \leftarrow \beta_1 \cdot c$$
$$\quad\quad M_n \leftarrow 0.85 \cdot f`_c \cdot b \cdot \beta_1 \cdot c \cdot \left(d - \frac{a}{2}\right) + A`_s \cdot E \cdot 0.003 \cdot \left(\frac{c - d`}{c}\right) \cdot \left(d - d`\right)$$
$$\text{else}$$
$$\quad M_n \leftarrow 0\ kip \cdot ft$$

$$\text{return } M_n$$

Moment Down Direction:

$$M_{n\_W} := M_n\left(A_{s\_W}, N_{bars\_W}, A`_{s\_W}, N`_{bars\_W}, W, T, d_W, d`_W, F_y, f`_c, E\right) = 51.427\ kip \cdot ft$$

$$M_{n\_L} := M_n\left(A_{s\_L}, N_{bars\_L}, A_{s\_L}, N`_{bars\_L}, L, T, d_L, d`_L, F_y, f`_c, E\right) = 38.141\ kip \cdot ft$$

$$\phi M_{n\_W\_Down} := 0.9 \cdot M_{n\_W} = 46.284\ kip \cdot ft$$

$$\phi M_{n\_L\_Down} := 0.9 \cdot M_{n\_L} = 34.327\ kip \cdot ft$$

$$P_u := \begin{bmatrix} 1.4 \cdot \left(P_{Dead} + P_{Self}\right) \\ 1.2 \cdot \left(P_{Dead} + P_{Self}\right) + 1.0 \cdot P_{Wind} \\ 0.9 \cdot \left(P_{Dead} + P_{Self}\right) + 1.0 \cdot P_{Wind} \end{bmatrix} = \begin{bmatrix} 11.9 \\ 3.2 \\ 0.65 \end{bmatrix} kip$$

Calculating Single Shear Demand:

$$V_{u\_W} := \text{if } \frac{Base_L}{2} + d_W < \frac{L}{2} \;\; \left\| \frac{P_u \cdot \left(\frac{L}{2} - \left(\frac{Base_L}{2} + d_L\right)\right)}{L} \right. \;\; \text{else} \;\; \left\| 0\ kip \right. = \begin{bmatrix} 2.913 \\ 0.783 \\ 0.159 \end{bmatrix} kip$$

$$V_{u\_L} := \text{if } \frac{Base_W}{2} + d_L < \frac{W}{2} \;\; \left\| \frac{P_u \cdot \left(\frac{W}{2} - \left(\frac{Base_W}{2} + d_W\right)\right)}{W} \right. \;\; \text{else} \;\; \left\| 0\ kip \right. = \begin{bmatrix} 3.925 \\ 1.056 \\ 0.214 \end{bmatrix} kip$$

Calculate Punching Shear Demand:

$$V_{u\_Punching} := \left\| \begin{array}{l} d \leftarrow min\left(d_W, d_L\right) \\ b_{o\_W} \leftarrow min\left(Base_W + d, W\right) \\ b_{o\_L} \leftarrow min\left(Base_L + d, L\right) \\ \text{if } b_{o\_W} \cdot b_{o\_L} > 0\ in^2 \\ \quad \left\| \text{return } P_u - \frac{P_u}{W \cdot L} \cdot \left(b_{o\_W} \cdot b_{o\_L}\right) \right. \\ \text{else} \\ \quad \left\| \text{return } 0\ kip \right. \end{array} \right. = \begin{bmatrix} 10.753 \\ 2.892 \\ 0.587 \end{bmatrix} kip$$

Calculate Factored Moment Demand:

$$M_{u\_W} := \frac{P_u \cdot \left(\frac{L}{2} - \frac{Base_L}{2}\right)^2}{2 \cdot L} = \begin{bmatrix} 3.347 \\ 0.9 \\ 0.183 \end{bmatrix} kip \cdot ft \qquad M_{u\_L} := \frac{P_u \cdot \left(\frac{W}{2} - \frac{Base_W}{2}\right)^2}{2 \cdot W} = \begin{bmatrix} 6.198 \\ 1.667 \\ 0.339 \end{bmatrix} kip \cdot ft$$

Check Footing Shear Capacity:

$$V_{u\_W} = \begin{bmatrix} 2.913 \\ 0.783 \\ 0.159 \end{bmatrix} kip \qquad V_{u\_L} = \begin{bmatrix} 3.925 \\ 1.056 \\ 0.214 \end{bmatrix} kip \qquad V_{u\_Punching} = \begin{bmatrix} 10.753 \\ 2.892 \\ 0.587 \end{bmatrix} kip$$

$$\phi V_{n\_W} = 36.971\ kip \qquad \phi V_{n\_L} = 24.648\ kip \qquad \phi V_{c\_punching} = 74.97\ kip$$

$$\frac{V_{u\_W}}{\phi V_{n\_W}} = \begin{bmatrix} 0.079 \\ 0.021 \\ 0.004 \end{bmatrix} \qquad \frac{V_{u\_L}}{\phi V_{n\_L}} = \begin{bmatrix} 0.159 \\ 0.043 \\ 0.009 \end{bmatrix} \qquad \frac{V_{u\_Punching}}{\phi V_{c\_punching}} = \begin{bmatrix} 0.143 \\ 0.039 \\ 0.008 \end{bmatrix}$$

$$shearDesignRatio := \max\left(\frac{V_{u\_W}}{\phi V_{n\_W}}, \frac{V_{u\_L}}{\phi V_{n\_L}}, \frac{V_{u\_Punching}}{\phi V_{c\_punching}}\right) = 0.159$$

Check Footing Moment Capacity:

$$M_{u\_W} = \begin{bmatrix} 3.347 \\ 0.9 \\ 0.183 \end{bmatrix} \; kip \cdot ft \qquad M_{u\_L} = \begin{bmatrix} 6.198 \\ 1.667 \\ 0.339 \end{bmatrix} \; kip \cdot ft$$

$$\phi M_{n\_W\_Down} = 46.284 \; kip \cdot ft \qquad \phi M_{n\_L\_Down} = 34.327 \; kip \cdot ft$$

$$\frac{M_{u\_W}}{\phi M_{n\_W\_Down}} = \begin{bmatrix} 0.072 \\ 0.019 \\ 0.004 \end{bmatrix} \qquad \frac{M_{u\_L}}{\phi M_{n\_L\_Down}} = \begin{bmatrix} 0.181 \\ 0.049 \\ 0.01 \end{bmatrix}$$

$$momentDesignRatio := \max\left(\frac{M_{u\_W}}{\phi M_{n\_W\_Down}}, \frac{M_{u\_L}}{\phi M_{n\_L\_Down}}\right) = 0.181$$

Bearing Check:

ASD Bearing Force: $P_{asd} = \begin{bmatrix} 8.5 \\ 1.5 \\ 0.9 \end{bmatrix} \; kip$ Resisting Force: $W_r = \begin{bmatrix} 8.5 \\ 8.5 \\ 5.1 \end{bmatrix} \; kip$

$bearingCheck :=$

```
i ← ORIGIN .. rows(P_asd) − 1
check ← 0
for j ∈ i
    if P_asd_j < 0 kip
        check_j ← −1 · (P_asd_j − W_r_j) / W_r_j
    else
        check_j ← P_asd_j / bearingCapacity_j
return check
```

$$= \begin{bmatrix} 0.118 \\ 0.021 \\ 0.013 \end{bmatrix}$$

Check For Uplift On Footing:

Service Level: $P_{asd} = \begin{bmatrix} 8500 \\ 1500 \\ 900 \end{bmatrix} lbf$ Strength Level: $P_u = \begin{bmatrix} 11900 \\ 3200 \\ 650 \end{bmatrix} lbf$

Note All values are positive, so no uplift from ASD Load Combinations

Force Resisting Strength Level Uplift Forces: $W_r := \begin{bmatrix} 1.4 \cdot (P_{Dead} + P_{Self}) \\ 1.2 \cdot (P_{Dead} + P_{Self}) \\ 0.9 \cdot (P_{Dead} + P_{Self}) \end{bmatrix} = \begin{bmatrix} 11.9 \\ 10.2 \\ 7.65 \end{bmatrix} kip$

Check for tension loads at top of footing:

$$P_t := P_u - W_r = \begin{bmatrix} 0 \\ -7000 \\ -7000 \end{bmatrix} lbf$$

Check Factored Uplift Shear:

$$V_{u\_W} := min\left(\frac{P_t}{2} - \frac{P_t \cdot (T - d`_L)}{L}\right) = -2.443\ kip$$

$$V_{u\_L} := min\left(\frac{P_t}{2} - \frac{P_t \cdot (T - d`_W)}{W}\right) = -2.795\ kip$$

Note: don't use the bearing plate dimensions for shear or moment capacity calculations because there's no base plate on the bottom of the footing.

Calculate Factored Punching Shear:

$$d_p := min\left((T - d`_W), (T - d`_L)\right) = 7.25\ in$$

$$V_{u\_Punching} := min\left(P_t - \frac{P_t}{W \cdot L} \cdot (d_p^{\ 2})\right) = -6.894\ kip$$

Calculate Factored Moment:

$$M_{u\_W} := min\left(\frac{L \cdot P_t}{8}\right) = -3.5\ kip \cdot ft \qquad M_{u\_L} := min\left(\frac{W \cdot P_t}{8}\right) = -5.25\ kip \cdot ft$$

Uplift Check:

```
upliftCheck := | i ← ORIGIN .. rows(P_u) − 1
               | check ← 0
               | for j ∈ i
               |   | if P_u_j − W_r_j < 0 kip
               |   |   | check_j ← (−1 · (P_u_j − W_r_j)) / W_r_j
               |   | else
               |   |   | check_j ← 0.0
               | return check
```

$$= \begin{bmatrix} 0 \\ 0.686 \\ 0.915 \end{bmatrix}$$

Single Shear Capacity - Up:

$V_{n\_W} := V_n\left(f`_c, W, T - d`_W, \rho_{\_W\_top}\right) = 57.182\ kip$ $\phi V_{n\_W} := 0.75 \cdot V_{n\_W} = 42.887\ kip$

$V_{n\_L} := V_n\left(f`_c, L, T - d`_L, \rho_{\_L\_top}\right) = 38.121\ kip$ $\phi V_{n\_L\_up} := 0.75 \cdot V_{n\_L} = 28.591\ kip$

$\phi V_{c\_punching\_up} := 0.75 \cdot V_{c\_punching}\left(W, L, 0\ in, 0\ in, min\left(T - d`_W, T - d`_L\right)\right) = 34.548\ kip$

$\text{if}\left(-1 \cdot V_{u\_W} \le \phi V_{n\_W}, \text{“OK”}, \text{“FAIL”}\right) = \text{“OK”}$ $-1 \cdot \frac{V_{u\_W}}{\phi V_{n\_W}} = 0.057$

$\text{if}\left(-1 \cdot V_{u\_L} \le \phi V_{n\_L}, \text{“OK”}, \text{“FAIL”}\right) = \text{“OK”}$ $-1 \cdot \frac{V_{u\_L}}{\phi V_{n\_L}} = 0.113$

$\text{if}\left(-1 \cdot V_{u\_Punching} \le \phi V_{c\_punching}, \text{“OK”}, \text{“FAIL”}\right) = \text{“OK”}$ $-1 \cdot \frac{V_{u\_Punching}}{\phi V_{c\_punching}} = 0.092$

Moment Capacity - Up Direction:

$M_{n\_W} := M_n\left(A`_{s\_W}, N`_{bars\_W}, A_{s\_W}, N_{bars\_W}, W, T, T - d`_W, T - d_W, F_y, f`_c, E\right) = 73.668\ kip \cdot ft$

$M_{n\_L} := M_n\left(A`_{s\_L}, N`_{bars\_L}, A_{s\_L}, N_{bars\_L}, L, T, T - d`_L, T - d_L, F_y, f`_c, E\right) = 54.122\ kip \cdot ft$

$\phi M_{n\_W} := 0.9 \cdot M_{n\_W} = 66.301\ kip \cdot ft$ $\phi M_{n\_L} := 0.9 \cdot M_{n\_L} = 48.71\ kip \cdot ft$

$\text{if}\left(-1 \cdot min\left(M_{u\_W}\right) \le \phi M_{n\_W}, \text{“OK”}, \text{“FAIL”}\right) = \text{“OK”}$ $-1 \cdot \frac{M_{u\_W}}{\phi M_{n\_W}} = 0.053$

$\text{if}\left(-1 \cdot min\left(M_{u\_L}\right) \le \phi M_{n\_L}, \text{“OK”}, \text{“FAIL”}\right) = \text{“OK”}$ $-1 \cdot \frac{M_{u\_L}}{\phi M_{n\_L}} = 0.108$

```
Footing Bearing Analysis:
    Bearing Code Check = 0.92
    P = -7.00 kips vs 7.65 kips
    For 0.9 Dead + 1.0 Wind
Footing Shear Analysis:
    Shear Code Check = 0.20
    Vu = -6.89 kips vs 34.55 kips
    For 0.9 Dead + 1.0 Wind
Footing Moment Analysis:
    Moment Code Check = 0.18
    Mu = 6.20 kip-ft vs 34.33 kip-ft
    For 1.4 Dead
```